Improving the Hydraulic Performance of Single Step Broad-Crested Weirs

Hamid H. Hussein [1] **Inam A. K. Juma** [2] **Saleh J. S. Shareef** [3]

Received: 15 February 2010; Accepted: 8 June 2010

Abstract

In this study the hydraulic performance of single step broad-crested weir was improved. Through analyzing the parameters that have effect on the shape of the step and its influence on the flow characteristics, and energy dissipation percent ($E\%$) downstream (D/S) of the weir. The differential equation of gradually varied flow for the water surface profile over the weir was solved analytically. Furthermore, empirical relations for $E\%$ and discharge coefficient (C_d) due to the affecting factors were derived .The results showed that the weir model when the ratio of the length of D/S step to the length of the weir L_2/L_1=0.5 gives a higher $E\%$ in comparison with other weir models. Three types of flow regimes were observed, nappe flow below 350 $cm^3/s/cm$, transition flow 350-700 $cm^3/s/cm$ and skimming flow upper than 700 $cm^3/s/cm$. The comparison between calculated values by the differential equation of gradually varied flow and experimental values gives a good agreement, the maximum difference is about 7%. Two empirical relations were obtained, the first to estimate Cd in terms of the ratio for upstream U/S water head to U/S weir height h/P_1 and L_2/L_1. While the second relation to estimate $E\%$ in terms of the ratio for D/S water head to U/S weir height h/P_1, L_2/L_1 and the Froude number Fr_2 with a high correlation coefficient.

Key Words: Hydraulics; weirs; performance; dissipation energy

تحسين الأداء الهيدروليكي للهدارات ذات القمة العريضة مفردة الدرجة

حامد حمود حسين **أنعام علي قاسم جمعه** **صالح جعفر سليمان شريف**

الخلاصة

في هذه الدراسة تم تحسين الأداء الهيدروليكي للهدار ذو القمة العريضة مفرد الدرجة من خلال تحليل المتغيرات التي تؤثر في شكل الدرجة الهندسي وتأثيرها على خصائص الجريان وتبديد الطاقة. كما تم حل المعادلة التفاضلية الخاصة بالجريان المتغير التدريجي لشكل سطح الماء فوق الهدار. بالإضافة الى ذلك اشتقت معادلات وضعية . أظهر تحليل النتائج المختبرية بأن الهدار(($L2/L1$=0.5 أعطى أعلى نسبة تبديد للطاقة بالمقارنة مع بقية الهدارات. لوحظ تكون ثلاثة أنواع للجريان فوق الهدار, النوع الأول هو الجريان المتدرج بتصريف أقل من ٣٥٠ سم٣/ ثا/ سم ,والثاني هو الجريان الانتقالي ما بين ٣٥٠- ٧٠٠ سم٣ / ثا/ سم , أما النوع الثالث فهو الجريان الانسيابي وكان التصريف أعلى من ٧٠٠ سم٣ / ثا/ سم . المقارنة بين النتائج المحسوبة للمعادلة التفاضلية الخاصة بالجريان المتغير التدريجي مع النتائج المختبرية أعطت تطابقا جيدا وأعلى اختلاف كان بنسبة ٧%. كما تم التوصل الى معادلتين وضعيتين. الأولى لحساب معامل التصريف C_d بدلالة ارتفاع الماء فوق قمة الهدار في المقدم الى ارتفاع الهدار في المقدم H/P_1 و طول الدرجة في المؤخر الى طول الهدار L_2/L_1. أما الثانية لحساب نسبة تبديد الطاقة $E\%$ بدلالة ارتفاع الماء فوق مؤخر الهدار الى ارتفاع الهدار في المقدم h/P_1 و L_2/L_1 ورقم فرود Fr_2 وبمعاملات ارتباط عالية..

---

[1] *Assistant professor at the Technical College - Mosul.*

[2] *Lecturer at the College of Engineering at the University of Mosul.*

[3] *Assistant lecturer at the Technical College - Mosul.*

1. *Introduction*

The flow characteristics over broad-crested weirs were investigated by several investigators, they have mainly concentrated on the overflow above these weirs. Some of the published papers that studied the flow characteristics of broad-crested weirs include [1];[2]; [3]; [4], presented the results of an experimental study of supercritical flow over drops. The authors of [5] studied in experimental measurements the velocity and pressure distributions over a large broad-crested weir, their results showed the rapid flow distribution at the upstream end of the weir ,while an overhanging crest design may affect the flow field. Also, they presented two empirical relations depend on the geometry of the weir. Dissipation of flow energy downstream (D/S) of hydraulic structures has been the interesting subject of many investigators. The authors of [6] focused on describing flows over small, stepped, homogeneous gabion spillways to quantify accurately the energy dissipation of standard designs and determine stilling-basin design parameters. The authors of [7] indicated that the stepped chute has been accepted to be the most powerful hydraulic structure to dissipate large flow energy downstream from steep hills. The authors of [8] proposed a relationship to express the relative energy dissipation in the presence and absence of boulders on the rock chutes. The authors of [9] investigated the relative energy dissipation in submerged flow conditions and proposed an exponential relationship which valid in different submerged conditions. The authors of [10] presented an experimental study on the hydraulics of flow through and over gabion-stepped weirs. They found that the energy loss in the gabion- stepped weirs were greater than those in the corresponding horizontal stepped weirs by approximately 7% to 14%. The authors of [11] changed the shape of traditional broad-crested weir by reducing D/S height of the weir to give it a new performance and make it as a water level controller and energy dissipater, the length of the D/S step was fixed, while the height of the D/S step was changed several times. The experimental results showed that the reduction in D/S height increased the energy dissipation percent up to 46%. Furthermore, the discharge coefficient was improved and gets higher values in comparison with traditional weirs.

This study is an extension to our previous study [11] by changing the D/S step length and fixing D/S step height. So that, the aim of this study is to investigate the effects of change the step geometry on the flow characteristics, and energy dissipation D/S of the stepped weir. Solve the differential equation of steady gradual flow for the water surface profiles over the weir. Also, derive empirical relations for the energy dissipation percent and discharge coefficient due to the affecting factors.

2. *Theoretical Analysis*

2.1 *Gradually-Varied Flow Equation*

The classic differential equation for steady gradual flow in open channels is ([12]; [13]):

$$\frac{dy}{dx}=\frac{S_o-S_f}{1-\alpha Q^2T/gA^3} \tag{1}$$

where:

y = depth of flow, cm
x = space coordinate in the flow direction, cm
S_0 = bed channel slope,
S_f = energy slope,
Q = discharge, cm^3/s
T = top width of the flow, cm
g = acceleration due to gravity, cm/s^2

A = cross sectional area, cm^2
α = velocity distribution coefficient.
For wide rectangular channel sections:
$q = Q/T$; $R = y$; $A = y\,T$; assume $\alpha = 1$; then equation (1) becomes:

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - q^2/gy^3} \tag{2}$$

since $y_c = \sqrt[3]{q^2/g}$
where y_c = critical depth of flow

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \left(y_c/y\right)^3} \tag{3}$$

For horizontal channel; $S_o = 0$, then

$$\frac{dy}{dx} = \frac{-S_f}{1 - \left(y_c/y\right)^3} \tag{4}$$

From Manning's equation

$$S_f = \frac{q^2 n^2}{y^{10/3}} \tag{5}$$

$$\frac{dy}{dx} = \frac{-q^2 n^2 / y^{10/3}}{1 - \left(y_c/y\right)^3} \tag{6}$$

Let: $k = -q^2 n^2$; $\lambda = y/y_c$; $d\lambda/dy = 1/y_c$; $dy = y_c\,d\lambda$
Substituting in equation (6)

$$\frac{y_c d\lambda}{dx} = \frac{k/(\lambda y_c)^{10/3}}{1 - (1/\lambda)^3} \tag{7}$$

$$\frac{d\lambda}{dx} = \frac{k/(y_c^{13/3}\lambda^{10/3})}{1 - 1/\lambda^3} \tag{8}$$

Let: $k_1 = k/y_c^{13/3}$; then

$$\frac{d\lambda}{dx} = \frac{k_1}{\lambda^{10/3}\left(1 - 1/\lambda^3\right)}$$

By indefinite integration;

$$k_1 dx = \left(\lambda^{10/3} - \lambda^{1/3}\right) d\lambda$$

$$x = \frac{1}{k_1}\left(\frac{3}{13}\lambda^{13/3} - \frac{3}{4}\lambda^{4/3} + c\right) \tag{9}$$

The value of "c" can be found from the boundary conditions, at $y = y_c$; $\lambda = 1$.

For the weir model of $L_2/L_1 = 0.5$ and run of q = 397 cm^2/s ; n = 0.011, at y_c , x = -17.25 cm then the value of "c" was found to be equal to 0.735 ; then equation (9) becomes

$$x = \frac{1}{k_1}\left(\frac{3}{13}\lambda^{13/3} - \frac{3}{4}\lambda^{4/3} + 0.735\right) \tag{10}$$

2.2 *Dimensional Analysis*

A- *Weir discharge coefficient, C_d*

The selected parameters that have an influence on the weir coefficient of discharge can be functionally expressed as follows

$$f_1\,(q, H, P_1, P_2, L_1, L_2, R, g) = 0 \qquad (11)$$

where:

q= discharge over the weir per unit width, cm^3/s / cm
H = U/S water head above the weir crest, cm
P_1= U/S weir height, cm
P_2 = D/S weir height, cm
L_1 = length of the weir, cm
L_2 = length of D/S step, cm
R = radius of U/S corner of the weir, cm
g = acceleration due to gravity, cm/s^2

The parameters in equation (11) may be expressed in nondimensional form as

$$C_d = \frac{q}{\frac{2}{3}H\sqrt{\frac{2}{3}gH}} = f_2\left(\frac{H}{P_1}, \frac{P_2}{P_1}, \frac{L_1}{P_1}, \frac{L_2}{L_1}, \frac{R}{P_1}\right) \qquad (12)$$

Since, P_1; P_2; L_1 and R are fixed in this study then equation (12) can be rewritten as

$$Cd = f_3\left(\frac{H}{P_1}, \frac{L_2}{L_1}\right) \qquad (13)$$

B- *Energy Dissipation Percent*

The energy dissipation percent (E %) due to the change in D/S step geometry can be expressed by the following functional relationship

$$E\% = f\,(q, P_1, H, h, P_2, L_1, L_2, g) \qquad (14)$$

where:

$E\%$= energy dissipation percent,
q= discharge over the weir per unit width, cm^3/s/cm
P_1= U/S weir height, cm
H = U/S water head above the crest, cm
h = D/S water head, cm
P_2 = D/S weir height, cm
L_1 = length of the weir, cm
L_2 = length of D/S step, cm
g = acceleration due to gravity. cm/s^2

By dimensional analysis

$$E\% = f_1\left(\frac{H}{P_1}, \frac{h}{P_1}, \frac{P_2}{P_1}, \frac{L_2}{L_2}, \frac{q^2}{g \cdot h^3}\right) \qquad (15)$$

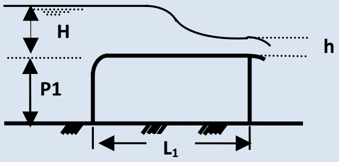
Since P_1 and P_2 are fixed and h value related to H and q^2/gh^3 is Froude number at Sec. (2) then equation (15) becomes

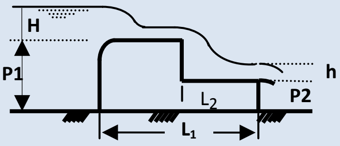
$$E\% = f_2\left(\frac{h}{P_1}, \frac{L_2}{L_1}, Fr_2\right) \tag{16}$$

3. *Experimental Setup and Procedure*

The stepped broad-crested weirs were installed in a horizontal, glass-walled rectangular channel, 0.50 m wide, 10 m long, and with 0.45 m height. Water was pumped from a laboratory sump to the V-notched tank from which water dropped to a dissipation tank with three dissipating baffles then water enters the horizontal channel (Plate 1). The discharge per unit width (q) varied from 100 to 800 cm^3 /s/cm, and it was measured by the V-notched weir tank. Upstream (U/S) water surface heads were started to measure at a location when $X/P_1 > 2$ U/S of the weir models as recommended by [11]. Seven weir models were made from thermo-stone and painted to decrease the surface roughness, the models were fixed at a distance of 3.5 m from the channel inlet. One of the weir models was a traditional round-nosed broad-crested weir, its length was 48 cm, 50 cm wide and 12 cm height with 6 cm radius of the round-nose. The other four models involved the above dimensions of the weir with D/S stepped height P2= 6 cm and D/S lengths L_2= 12, 18, 24 and 30 cms. While in the last two models the length of the weirs were increased to 54 and 60 cms with D/S lengths 30 and 36 cms respectively, as shown in Table 1. Each test included the measurements (under free flow conditions) for water surface levels, U/S flow depth, D/S depth over the step, and water discharge.

Table* 1. *Details of the tested weir models.





Model No.	L_1 (cm)	L_2/L_1	Run No.	H (m)	q (cm^2/sec)	C_d	Fr
1	L_1=48 L_2=48	1	1-7	0.157-0.215	270 -595	0.968-0.988	4.3-4.6
2	L_1=48 L_2=30	0.625	8-14	0.138-0.241	196 -794	0.897-0.952	5.5-5.6
3	L_1=48 L_2=24	0.5	15-21	0.12-0.22	139-631	0.954-0.988	4.7-6.1
4	L_1=48 L_2=18	0.375	22-28	0.142-0.228	211-690	0.845-0.879	5.2-7.2
5	L_1=48 L_2=12	0.25	29-35	0.105-0.234	100-737	0.853-0.923	7.6-8.2
6	L_1=54 L_2=30	0.56	36-42	0.115-0.232	125-721	0.907-0.954	5.3-6.3
7	L_1=60 L_2=36	0.6	43-49	0.159-0.233	279-729	0.869-0.915	1.8-2.1

Plate 1. Photograph showing the laboratory horizontal channel with V-notched tank and dissipation tank.

4. *Results and Discussion*

Three types of flow regimes were observed in all tests, that was, nappe flow, transition flow, and skimming flow. A free-falling nappe was found at small discharge (< 350 $cm^3/s/cm$) when the flow depth is smaller than step height. For transition flow at intermediate discharge (350-700 $cm^3/s/cm$), the free-falling disappears and the water surface was wavy, and flow depth a little pit less than step height. While for skimming flow at large discharge (> 700 $cm^3/s/cm$) the water surface was smooth and no nappe was visible. Furthermore, the step was submerged beneath a strong current (Plates 2, 3 and 4).

Figures 1, 2, and 3 present the variation of nondimensional water surface profiles (w.s.ps) for different discharges over three models of stepped weirs, where L_2/L_1 = 0.25, 0.375 and 0.625 respectively. In all of these figures w.s.ps. tends to reduce gradually when the water approach the weirs. Once the water across the crest of the weir the water passed rapidly and the flow changed from gradually varied flow to rapidly varied flow. The measured values of w.s.ps. and the calculated values by equation (10) over different stepped weir models ,for different discharges, were tested but the nearest with a good matching when L_2/L_1 =0.5 ,and used to plot in Figure 4. The plotted area for the measured and calculated values were shown in the reach of gradually varied flow. In this reach the comparison between both w.s.ps. gives a good agreement and the maximum difference is about 7% .

Plate 2. Flow over stepped weirs.

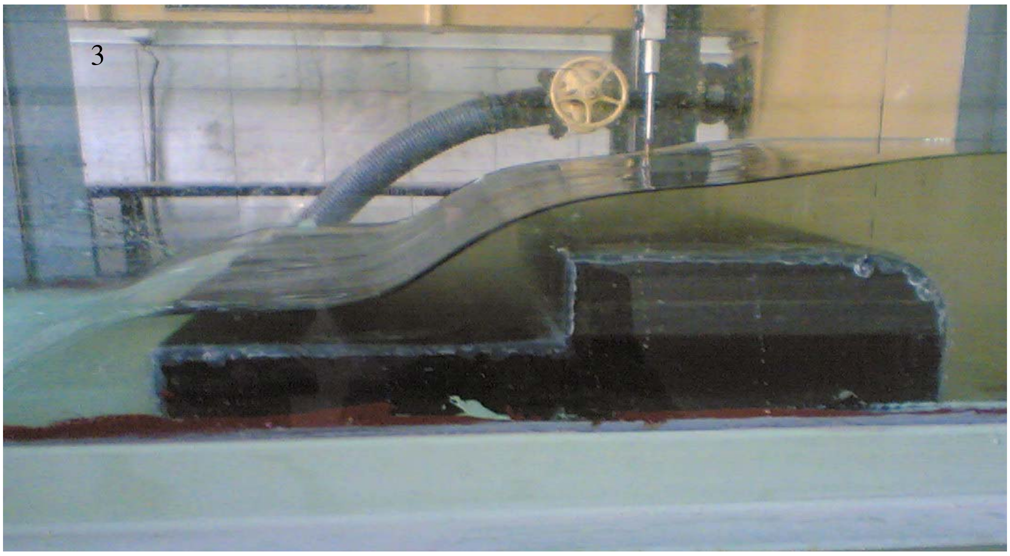


Plate 3. Nappe flow.

Plate 4. Skimming flow.

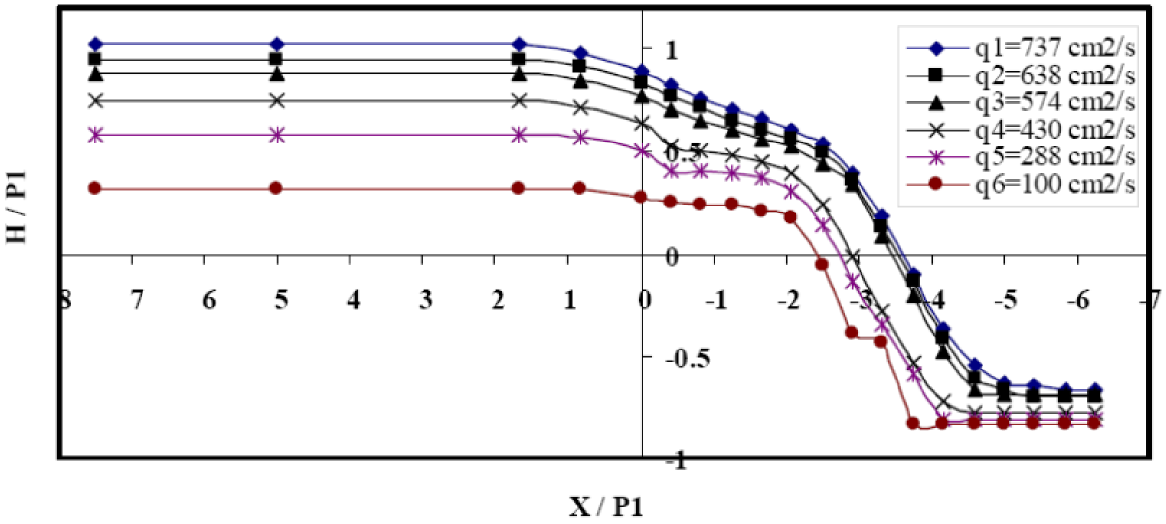


Figure 1. Dimensionless water surface profile for weir model of L_2/L_1*=0.25.*

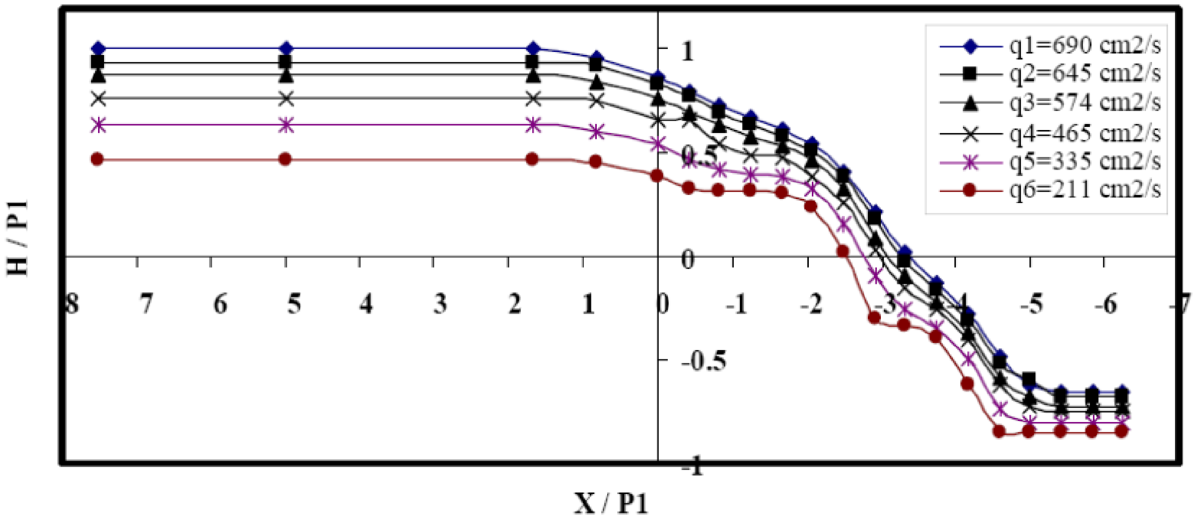


Figure 2. Dimensionless water surface profile for weir model of L_2/L_1*=0.375.*

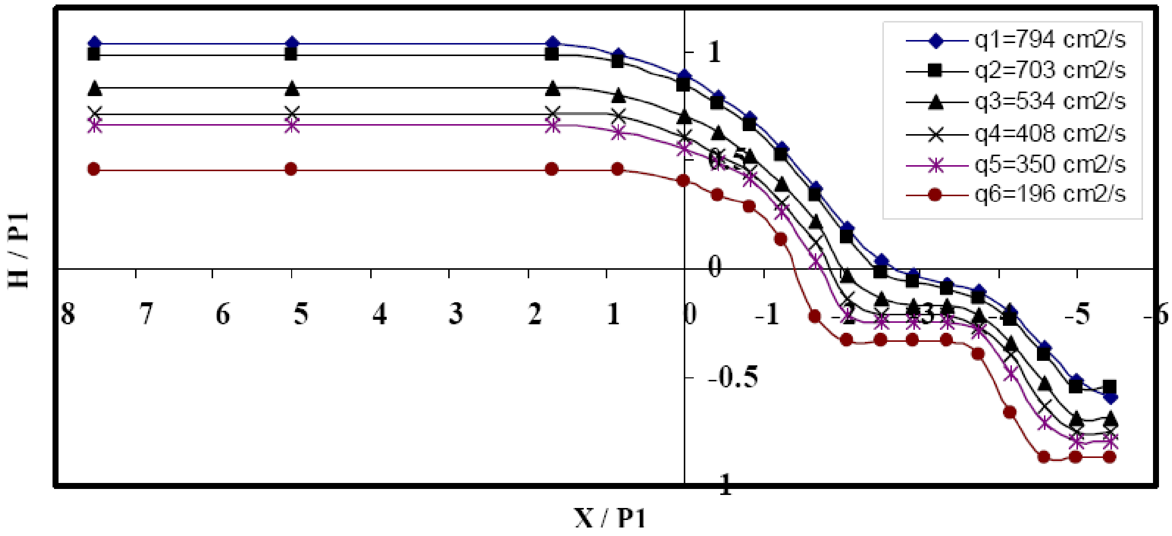


Figure 3. Dimensionless water surface profile for weir model of L_2/L_1*=0.625.*

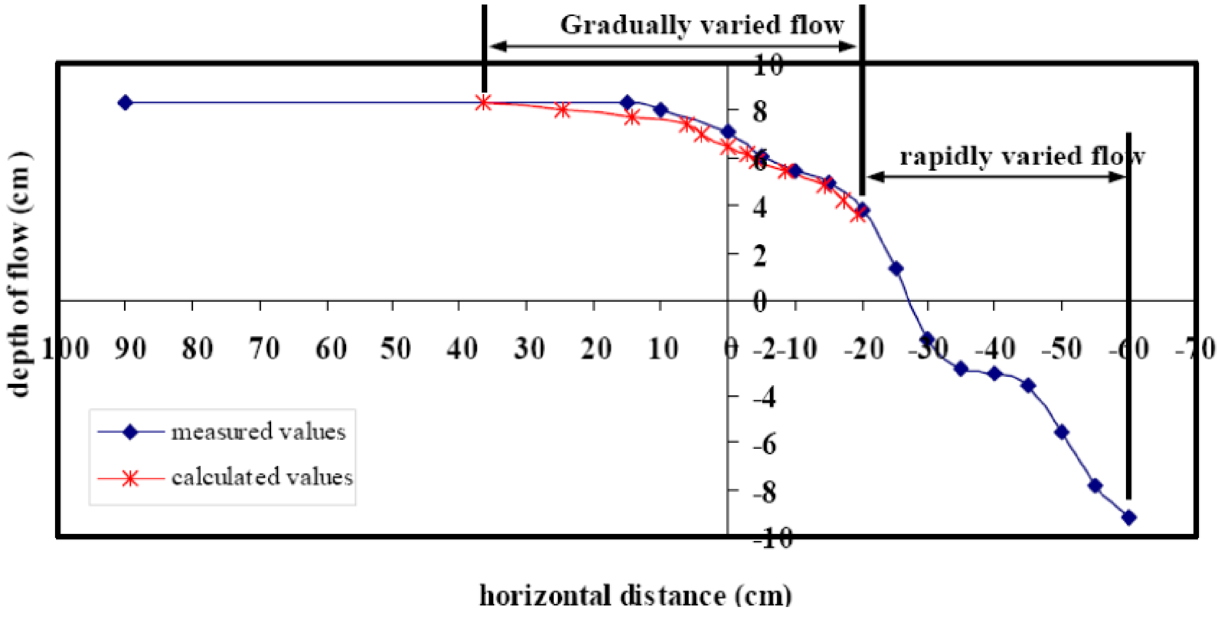


Figure 4. *Comparison between calculated values with measured ones for the water surface profile over stepped weir for L_2/L_1=0.5.*

Figure 5 depicts the variation of the energy dissipation percent $E\%$ with the Froude number Fr_2 for different stepped weirs. It was shown that $E\%$ increases as Fr_2 increases .On the other hand, the weir model when L_2/L_1=0.5 gives a higher $E\%$ in comparison with other weir models, for example when Fr_2 = 6 the values of $E\%$ for weir models 0.25, 0.625 and 0.5 were 19%, 22% and 34% respectively.

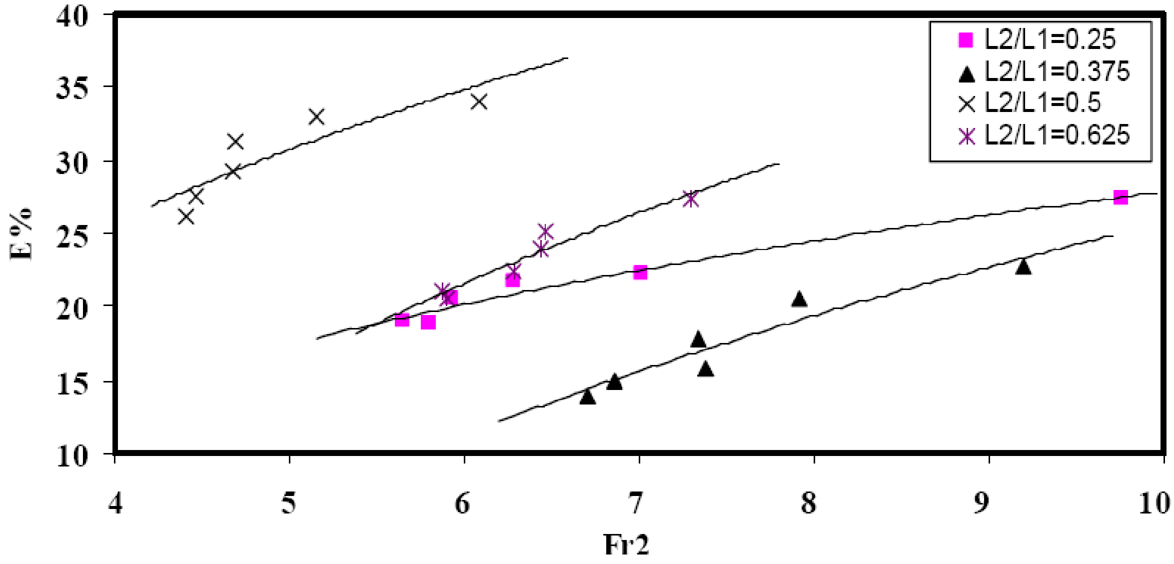


Figure 5. *Variation of E% with Fr_2 for different step weirs.*

Effect of the nondimensional parameter h/P_1 on $E\%$ for different weir models were shown in Figure 6, it was observed that $E\%$ decreased gradually with the increase of h/P_1. Also, it was observed that $E\%$ for the traditional weir model (*i.e.*, L_2/L_1=0) almost remains constant with the increase of h/P_1. While $E\%$ at other weir models was decreased clearly, for example , when h/P_1= 0.4 the values of $E\%$ for weir models 0.375, 0.25, 0.625 and 0.5 were 15%, 21%, 24% and 30% respectively.

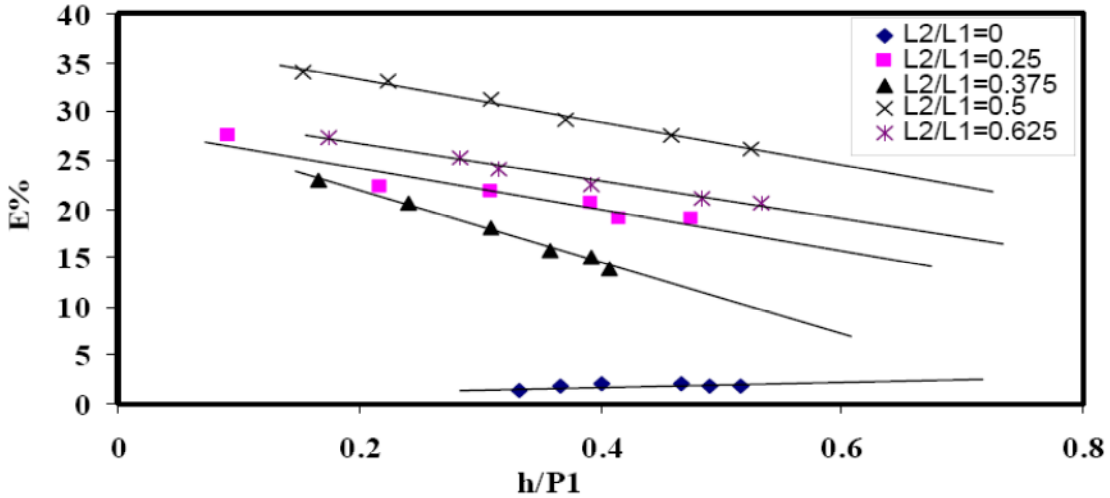


Figure 6. Dimensionless water surface profile for weir model of L_2/L_1=0.25.

The variation of C_d with H/P_1 was shown in Figure 7, where C_d was found to increase with the increase of L_2/L_1 ,as observed in the figure the highest values of C_d when L_2/L_1= 0.625.

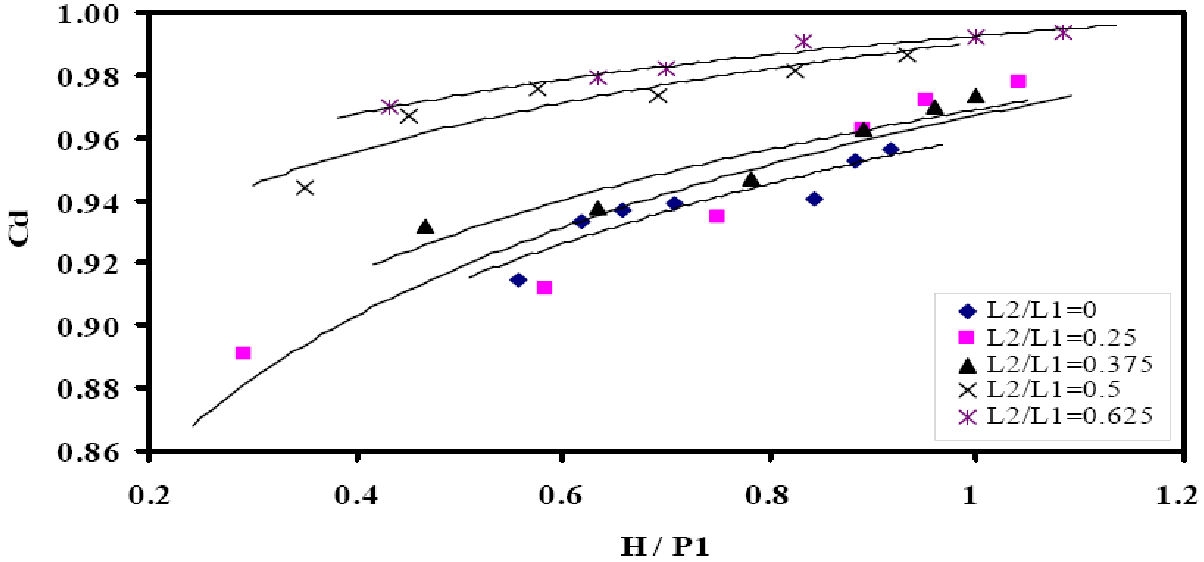


Figure 7. Relation between C_d and H/P_1 for weirs.

5. *Empirical Relations*

Based on equation (13), multiple nonlinear regression analysis was used to correlate C_d with both (H/P_1) and (L_2/L_1) in an empirical power relation

$$Cd = 1.02\left(\frac{H}{P_1}\right)^{0.05}\left(\frac{L_2}{L_1}\right)^{0.41} \tag{17}$$

with a correlation coefficient of 0.89.

The relation between C_d values predicted by equation (17) and experimental values of C_d is plotted in Figure 8 showing a good agreement.

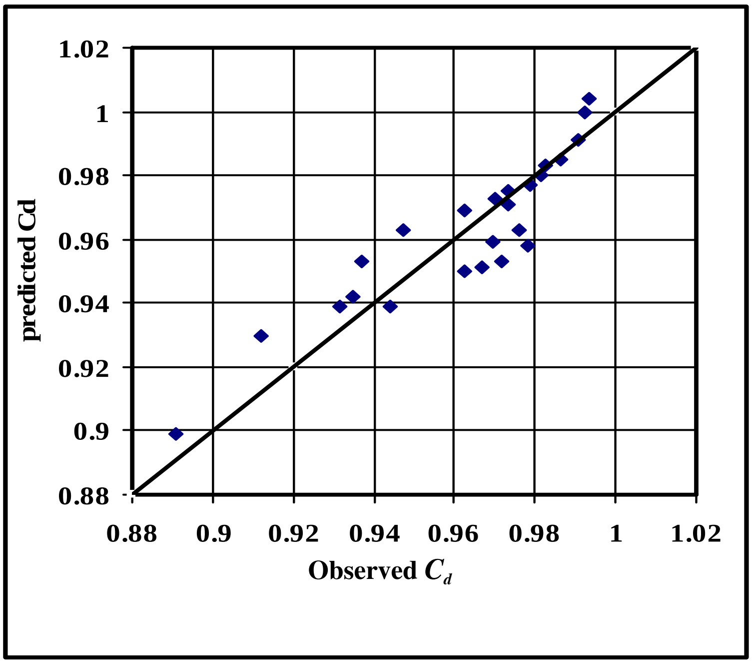


Figure 8. Variation of predicted values of C_d with the observed ones for all models.

Another empirical power relation based on equation (16) was obtained for the variation of *E%* with (h/P$_1$), (L_2/L_1) and Fr_2

$$E\% = 24.4\left(\frac{h}{P_1}\right)^{-0.57}\left(\frac{L_2}{L_1}\right)^{0.02} Fr_2^{-1.24} \tag{18}$$

with a correlation coefficient of 0.96.

A comparison between *E%* values predicted by equation (18) and observed values experimentally is shown in Figure 9 and showing a good agreement.

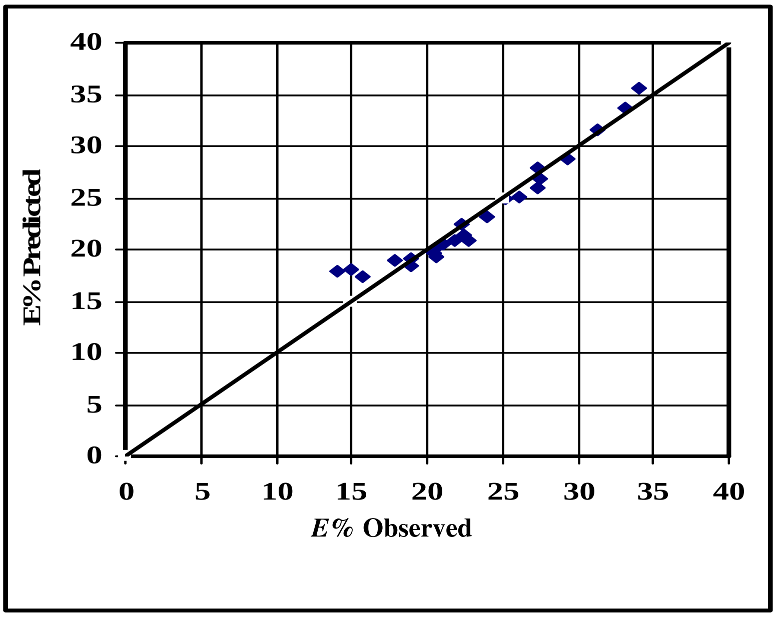


Figure 9. Variation of predicted values of C_d with the observed ones for all models.

6. *Conclusions*

Depending upon the results and analysis of this study the following conclusions are summarized below:

1. The weir model when L_2/L_1=0.5 gives a higher *E%* in comparison with other weir models. So, it can be recommended to use this weir in the design of prototype weirs.
2. Three types of flow regimes were observed, nappe flow below 350 cm^2 /s, transition flow 350-700 cm^2/s and skimming flow upper than 700 cm^2/s.
3. Over single step weirs there are two parts of flow, in the first part the flow is Gradually-varied flow while in the second part the flow is rapidly- varied flow.
4. A direct solution for the equation of gradually-varied flow at horizontal channel was derived, the comparison between calculated and experimental values gives a good agreement and the maximum difference is about 7%.
5. Two empirical relations were obtained, the first to estimate C_d in terms of H/P_1 and L_2/L_1 as shown in Equation (17). While the second relation to estimate *E%* in terms of h/P_1, L_2/L_1 and Fr_2 as shown in Equation (18).

References

1. Ackers, P.; White, W.R.; Perkins, J. A. and Harrison, A. J. M., 1978, *Weirs and Flumes for Flow Measurement,* John Wiley and Sons, Inc., New York, N.Y., 133-166..
2. Ramamurthy, A. S.; Tim, U.S. and Rio, M. V. J., 1988, “Characteristics of Square-Edged and Round- Nosed Broad-Crested Weirs”, *Journal of Irrigation and Drainage Engineering, ASCE*, **99**(2), 61-73.
3. Chamani, M. R. and Beirami, M. K., 2002, “Flow Characteristics at Drops”, *Journal of Hydraulic Engineering, ASCE*, **128**(8), 788-791.
4. Noori, B. M. and Juma, I. A., 2007, “Performance improvement of broad-crested weirs", *Journal of Al-Rafidain Engineering*, Mosul University, **17**(2), 95-107.
5. Gonzalez, C. A. and Chanson, H., 2007, “Experimental Measurements of Velocity and Pressure Distributions on a Large Broad-Crested Weir”, *Flow Measurement and Instrumentation,* Vol. 18, pp. 107-113.
6. Peyras, L. ,Royet, P., and Degoutte, G., 1992, “Flow and energy dissipation over stepped gabion weirs.” *Journal of Hydraulic Engineering, ASCE*, **118**(5), 707-717.
7. Chinnarasri, C. and Wong wises, S., 2006, “Flow patterns and energy dissipation over various stepped chutes." *Journal of Irrigation and Drainage Engineering, ASCE*, **132**(1), 70-76.
8. Pagliara,S., and Chiavaccini, P., 2006, “Energy dissipation on reinforced block ramps.” *Journal of Irrigation and Drainage Engineering, ASCE*, **132**(3), 293-297.
9. Pagliara, S., Das, R., and Palermo, M., 2008, “Energy dissipation on submerged block Ramps.” *Journal of Irrigation and Drainage Engineering, ASCE*, **134**(4), 527- 532.
10. Chinnarasri, C., Donjadee, S., and Israngkura, U., 2008, “Hydraulic characteristics of gabion-stepped weirs." *Journal of Hydraulic Engineering, ASCE*, **134**(8), 1147-1152.
11. Hussein, H. H., Juma, I. A., and Shareef, S. J., 2009, “Flow characteristics and energy dissipation over stepped round – nosed broad –crested weirs" , accepted in *Journal of Al-Rafidain Engineering.*, Mosul University.
12. Chow, V. T., 1959, *Open-channel Hydraulics*, McGraw-Hill, New York.
13. Venutelli, M., 2004, “Direct integration of the equation of gradually varied flow.” *Journal of Irrigation and Drainage Engineering, ASCE*, **130**(1), 88-91.