Nonlinear Analysis of Reinforced Concrete Slabs at Elevated Temperature

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Received: 3 June 2010; Accepted: 7 October 2010

Abstract

In this paper a nonlinear finite element analysis is presented to simulate the fire resistance of reinforced concrete slabs at elevated temperatures. An eight node layered degenerated shell element utilizing Mindlin/Reissner thick plate theory with initial stiffness technique is employed. The proposed model considered cracking, crushing, and yielding of concrete and steel at high temperatures. More complicated phenomena like concrete transient thermal strain and concrete spalling are excluded in the present analysis. The validation of the proposed model is examined against experimental data of previous researches and shows good agreement.

Key Words: Fire resistance, Material nonlinearity, Reinforced Concrete Slabs

التحليل اللاخطي للسقوف الخرسانية المسلحة عند الدرجات الحرارية العالية

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الخلاصة

استخدم التحليل اللاخطي بطريقة العناصر المحددة لمحاكاة مقاومة السقوف الخرسانية المسلحة المتعرضة لدرجات حرارة عالية (الحرائق)، مستخدما نظرية الصفائح السميكة مع تقنية الصلابة الابتدائية. تضمن النموذج المقترح في الدراسة حالة التشقق والسحق للخرسانة وكذلك حالة الخضوع لحديد التسليح ، العديد من الظواهر المعقدة مثل ظاهرة انتقال انفعال الحرارة للخرسانة لم تشمل في الدراسة الحالية، تم مقارنة نتائج النموذج المقترح مع العديد من الدراسات السابقة واظهر النموذج توافقا جيدا مع النتائج العملية للدراسات السابقة.

1. Introduction

The Research into the effect of fire on concrete, steel, and reinforced concrete structures has been conducted since at least 1922 (as shown by Schneider [1]). The main areas of interest are [2]: the understanding of the complex behaviour of the material itself, and the structural safety and integrity of the building during and after the fire. The analytical prediction of reinforced concrete structures at high temperatures back to 1970s. The thermal and structural analysis are interfaced and not integrated. In other words, the temperatures distribution within the structural element due to temperature rise is first calculated for the entire duration of temperature exposure, and then fed into the structural analysis program to produce the stresses and strains for the structures [2].

The Nizamuddin and Bresler [3] presented a nonlinear finite element computer program FIRES-SL for the analysis of reinforced concrete slabs at fire. The analysis based on Kirchhoff thin plate theory and includes the coupling of bending and membrane action. Borst and Peeters [4] developed an algorithm which simultaneously considers the effects of thermal dilatation, degradation of the

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elastic properties with increasing temperature, transient thermal strain, and smeared cracking to simulate plain and reinforced concrete at high temperatures. Results of the presented model show a good agreement with experimental tests for both plain and reinforced concrete models. Huang et al [5] developed a nonlinear finite element approach to enable practical modelling of fairly large composite steel/concrete frames under the influence of compartment fire and external loading. Again the validity of the proposed models show a good agreement with experimental results.

Ahmed and Al-Zubaedi [6] used the finite difference method to study the nonlinear structural behaviour of reinforced concrete plates. Large deflection theory and dynamic relaxation technique were used to calculate the strains and stresses in the plate at elevated temperature.

Ahmed and Hasan [7] study the effect of cyclic heating and cooling on the nonlinear behaviour of reinforced concrete slabs at different load conditions before and after cracking up to failure. Finite difference method with dynamic relaxation technique was used in the solution of the differential equations.

The structural response involves the behaviour of reinforced concrete slabs which it's constitutes were affected by temperature rise under the applied load. In the present work, the combined effect of the two responses is systematically evaluated using computer software presented by Owen and Figueiras [8]. The software is modified in order to accommodate the evaluation of temperature exposure as well as the structural response. Transient thermal strain which is a function of temperature rise and applied load [9] and greatly affected the overall response of concrete slabs at high temperature, and concrete spalling which can expose steel reinforcement to much higher temperatures, causing further degradation of slab strength and stiffness, these two phenomena were especially excluded from the present research.

2. *Problem Material Constitutive Relationships at Elevated Temperature*

2.1 *Biaxial Concrete State*

At present, there is still little data and few theoretical models available concerning the constitutive modelling of concrete under biaxial stress state at elevated temperatures [5]. Therefore in this paper the failure envelope at ambient temperature is extended to elevated temperature taking into account the changes into its related parameters with temperature rise. So at each temperature level there is a failure envelope having the same characteristics of the failure envelope at ambient temperature. The formulation of the failure envelope proposed by Hinton and Owen [8] which is slightly differs from Kupfer *et al*. [10] was adopted in the present study.

2.2 *Uniaxial Compressive Strength of Concrete*

As shown by many researches, the concrete compressive strength deteriorate with temperature rise, the rate of deterioration varies considerably with temperature levels. The model in Figure 1 is used in the present study. Concrete is assumed to loss all stiffness after crushing.

2.3 *Modulus of Elasticity*

At elevated temperature a decrease in modulus of elasticity is mostly affected by the aggregate types [11]. The relationship between the modulus of elasticity and temperatures rise used in the present study shown in Figure 2 and used by Said [12] and indicates almost a constant rate of decrease in elasticity with increasing temperatures.

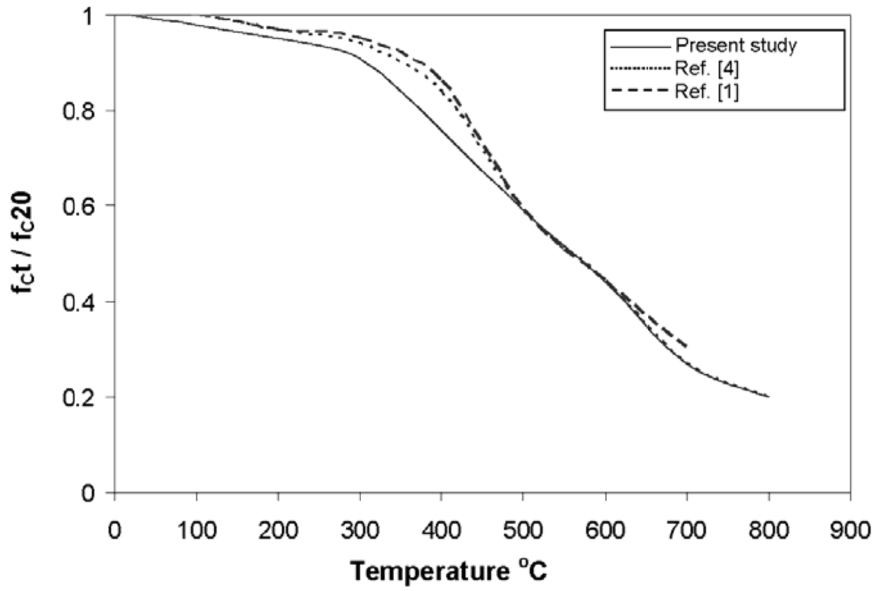

Figure 1. Variation of concrete compressive strength with temperature rise.

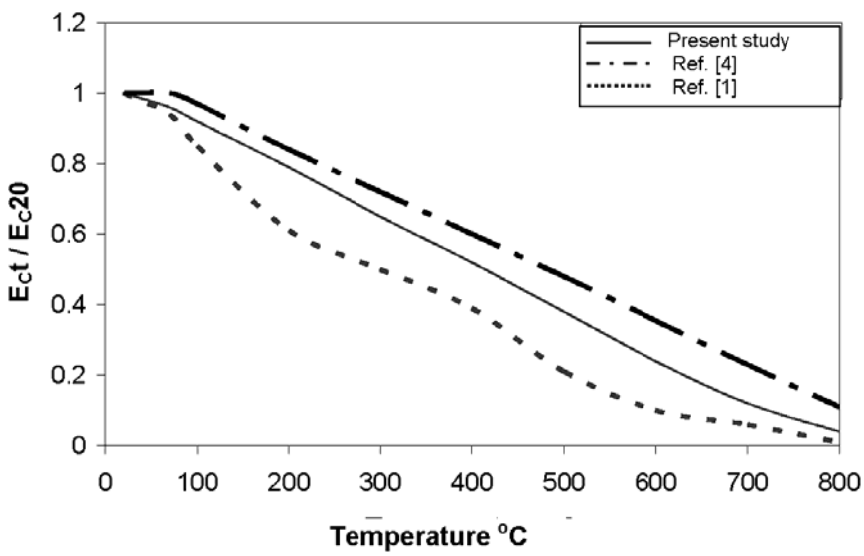

Figure 2. Variation of modulus of elasticity with temperature rise.

2.4 *Tensile Strength*

There is paucity of experimental date with regard to the dependence of concrete tensile strength with temperature, Figure 3 shows the model used in the analysis. Furthermore a gradual release of the concrete stress component normal to the cracked plane has been used to model the tension stiffening phenomena [8]. Unloading and reloading of cracked concrete is assumed to follow the linear behaviour shown with a fictitious elasticity modulus E_i given by

$$E_i = \alpha \ f_t^{'}(1-\varepsilon_i / \varepsilon_m) / \varepsilon_i \qquad \varepsilon_t \le \varepsilon_i \le \varepsilon_m$$

where (α, ε_m) are parameters of tension stiffening,

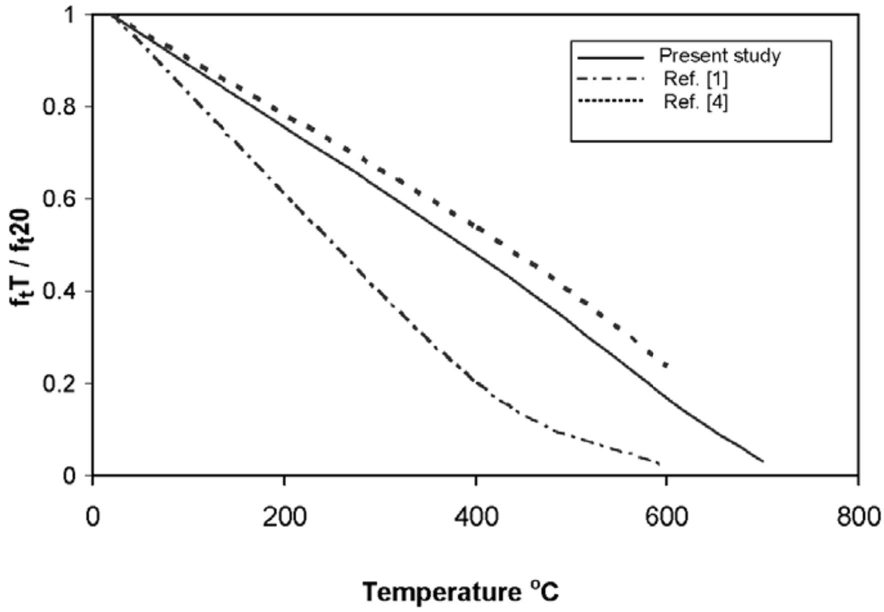

Figure 3. Variation of concrete tensile strength with temperature rise.

2.5 *Poisson's Ratio*

As reported by many researches [1], the results of relation between Poisson's ratio and temperature increase are erratic and no general trend of effect of temperature was clearly evident. As a result a constant value of 0.19 of Poisson's ratio at all temperature levels is adopted in the present research.

2.6 *Concrete Strain*

The failure of concrete may be determined by the maximum of attainable compressive strain $\varepsilon \le \varepsilon_u$ which have been established in a total deformation test. A single unique relation between strain and temperature reported by Schneider [1] as shown in Figure 4 is used by Said [12] and adopted in the present study.

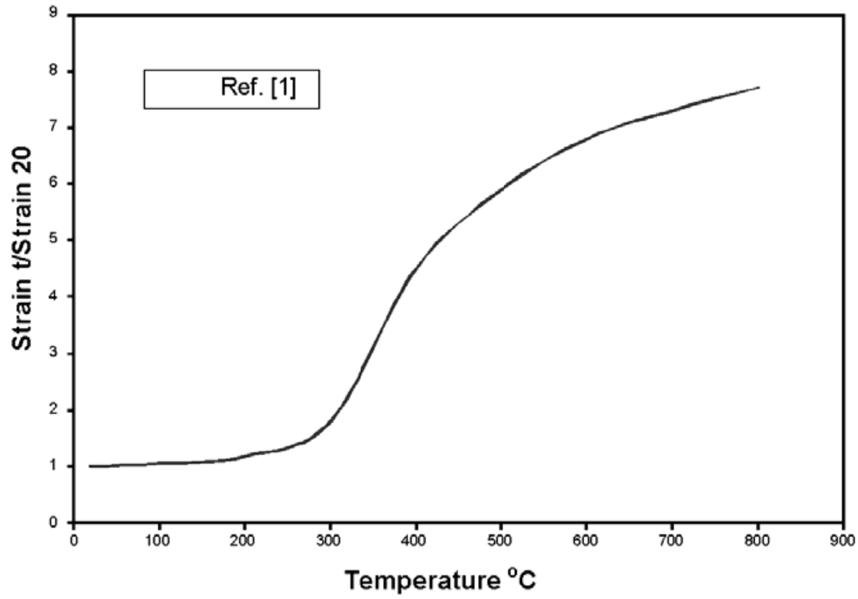

Figure 4. Variation of Strain with Temperature Rise.

2.7 Steel Reinforcement

The reinforcing bars are considered as steel layers of equivalent thickness in the present study. Each steel layer has a uniaxial behaviour resisting only the axial force in bar direction. The properties of reinforcement (yield strength and modulus of elasticity) are presented in Figures 5 and 6 [13].

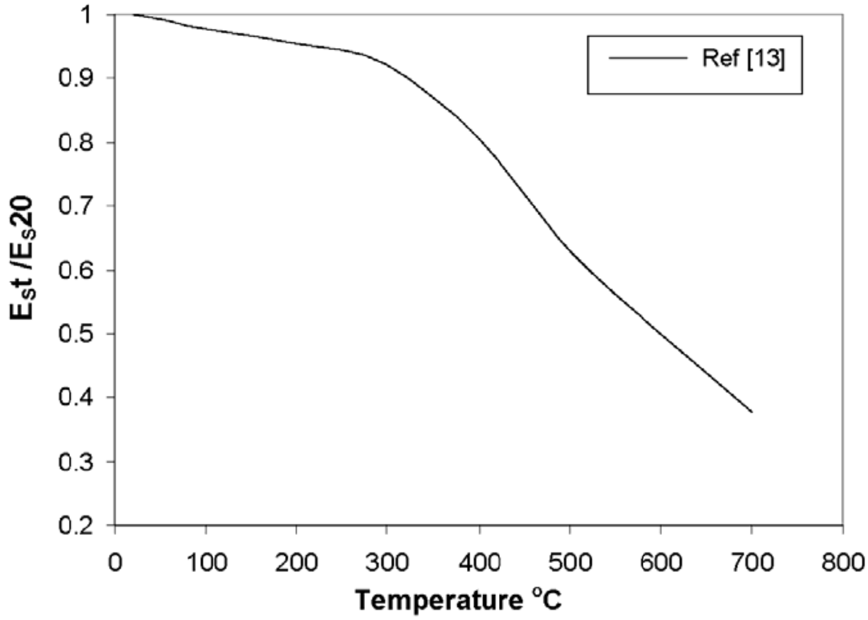

Figure 5. Variation of steel modulus with temperature rise.

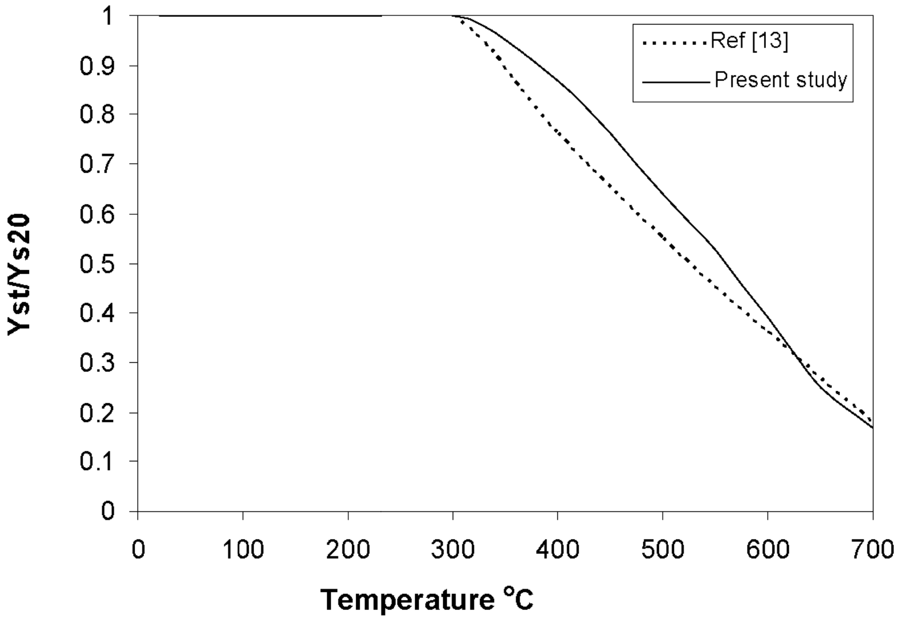

Figure 6. Variation of steel yield strength with temperature rise.

3. Structural Response and Finite Element Discretisation

Structural response is defined in terms of transient deformations, forces, moments, and material degradation (cracking, crushing, and yielding) throughout the slab. These factors account for material behavior at varying temperatures, including dimensional changes caused by temperature variation, changes in mechanical properties with changes in temperature, degradation of section through cracking and crushing, and inelastic deformations associated with nonlinear stress, strain characteristics. An eight node thick shell Ahmad element with layer model which subdivide the slab into concrete and steel layers to allow the consideration of temperature variation and the consequent materials properties through slab depth. Slabs are modeled as an assemblage of finite elements connected at nodal points, each element is considered to be made up of number of layers representing plain concrete or reinforcing steel. Each layer can have a different temperature, but with same layer the temperature is constant, therefore material properties within each layer is uniform. The basic analytical problems is to determine the deformation history of the nodes $r(t)$ when external loading at the nodes $R(t)$ is applied while the temperature increases. A finite element method using nonlinear initial stiffness formulation coupled with time step integration is used to predict the structural response as shown in Figure 7.

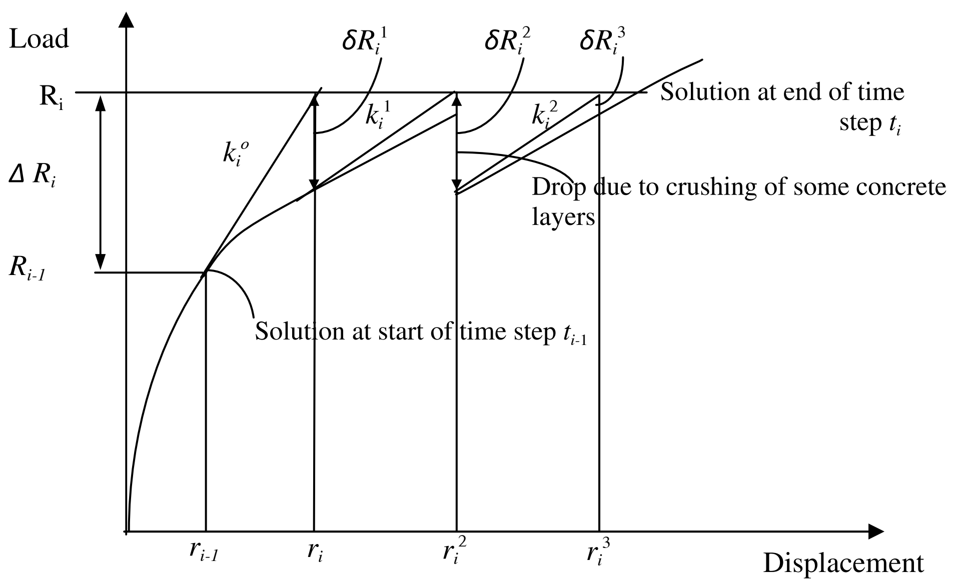

Figure 7. Solution of time dependent nonlinear equilibrium equation.

A time step begins when a time increment Δt_i and external load increment ΔR_i are read. These values are added to the previous time and load values to obtain the current time t_i and load vector R_i. The first iteration in a time step is initiated with the assumption that the current deformation r_i is equal to the deformation at the end of the previous time step r_{i-1}. The current tangent stiffness k_i^j and the current internal forces are calculated. If equilibrium not satisfied *i.e.*, the difference between the external applied load and the internal forces greater than the permissible tolerance a correction deformation is calculated and added to the current deformation and thereby a more accurate deformation shape is determined. The iterative cycle is repeated, a new tangent stiffness and internal forces is calculated on the basis of the new deformation shape. Iteration continues until equilibrium is satisfied within the permissible tolerance.

4. *Numerical Verification*

For validation of the proposed model, a simply supported (4900 × 1900 mm) reinforced concrete slab subjected to fire test at University of Gent and analyzed by Brost and Peeters [4] is considered here. The slab was simply supported on the short sides while the displacements at the other two sides were free. The slab thickness is 150 mm loaded by two jacks over the full width. Slab geometry, loading, and material properties are summarized in Table 1. Only one-quarter of the slab was analyzed since both slab and them heated area were symmetrical. The slab response under the applied load at fire environment is presented in terms of time-deflection. As shown in Figure 9, there is good agreement between the results of the present study with experimental results especially from time 25 minutes until failure. But the differences between the present analysis and the experimental data are larger than that between [4] and experimental results especially at the first stage of heating and this can attributed to the neglecting of geometrical nonlinearity analysis and transient thermal strain in the present analysis which proved to take major part during the first heating of concrete [2].

Table 1. Material properties and reinforcement of the slab in Figure 8.

E MPa	f_c MPa	f_t MPa	υ	α $^{o}C^{-1}$	E_s MPa	f_{sy} MPa	Reinforcement (mm^2)	Self-weight kN/m	Load weight kN/m
45200	42.7	2.6	0.2	0.000012	215000	504	1178	6.29	14.5

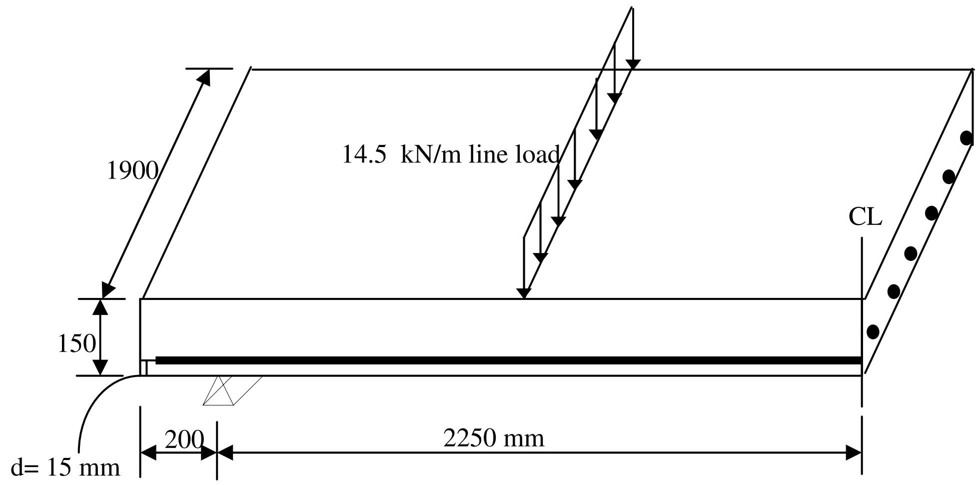

Figure 8. The slab analysed by Brost and Peeters.

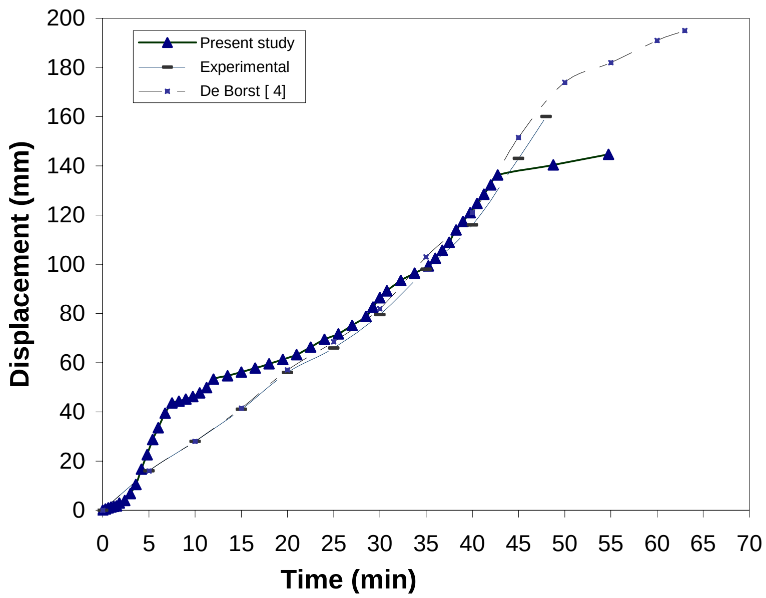

Figure 9. Comparison between experimental and predicted time-deflection.

Another comparison made with full scale fire test on slab specimen tested by the PCA [3] as a part of a research programme on the response of reinforced concrete slabs at fire. The slab was 2.7 m square, 100 mm thick Figure 10 reinforced with 10 mm steel bars top and bottom spaced at 150 mm in both directions. Slab material properties are shown in Table 2. The central (840 mm × 840 mm) area of the slab was exposed to the ASTM E119 standard fire, the remaining portion of the slab was outside the furnace. The slab was supported vertically near the corners of the heated area while the outer edges were free. Since both the slab and the heated area were symmetry, only one-quarter of the slab is analyzed in the present analysis. The variation of central deflection with time is shown in Figure 11. Differences between the experimental results and the present analysis are quite large especially at first stage of heating until 0.3 hour, after which, good agreement between the results of the present study and the experimental results, is found.

The difference at the first stage of heating could be related to the constitutive models adopted in the present analysis especially the tensile strength model which greatly influences the response of concrete at high temperature under applied load.

Table 2. Material properties at ambient temperature of the slab in Figure 10.

E (MPa)	f_c (MPa)	f_t (MPa)	υ	α °C^{-1}	E_s(MPa)	f_{sy} (MPa)
27590	32.35	5.17	0.19	0.000012	20700	415

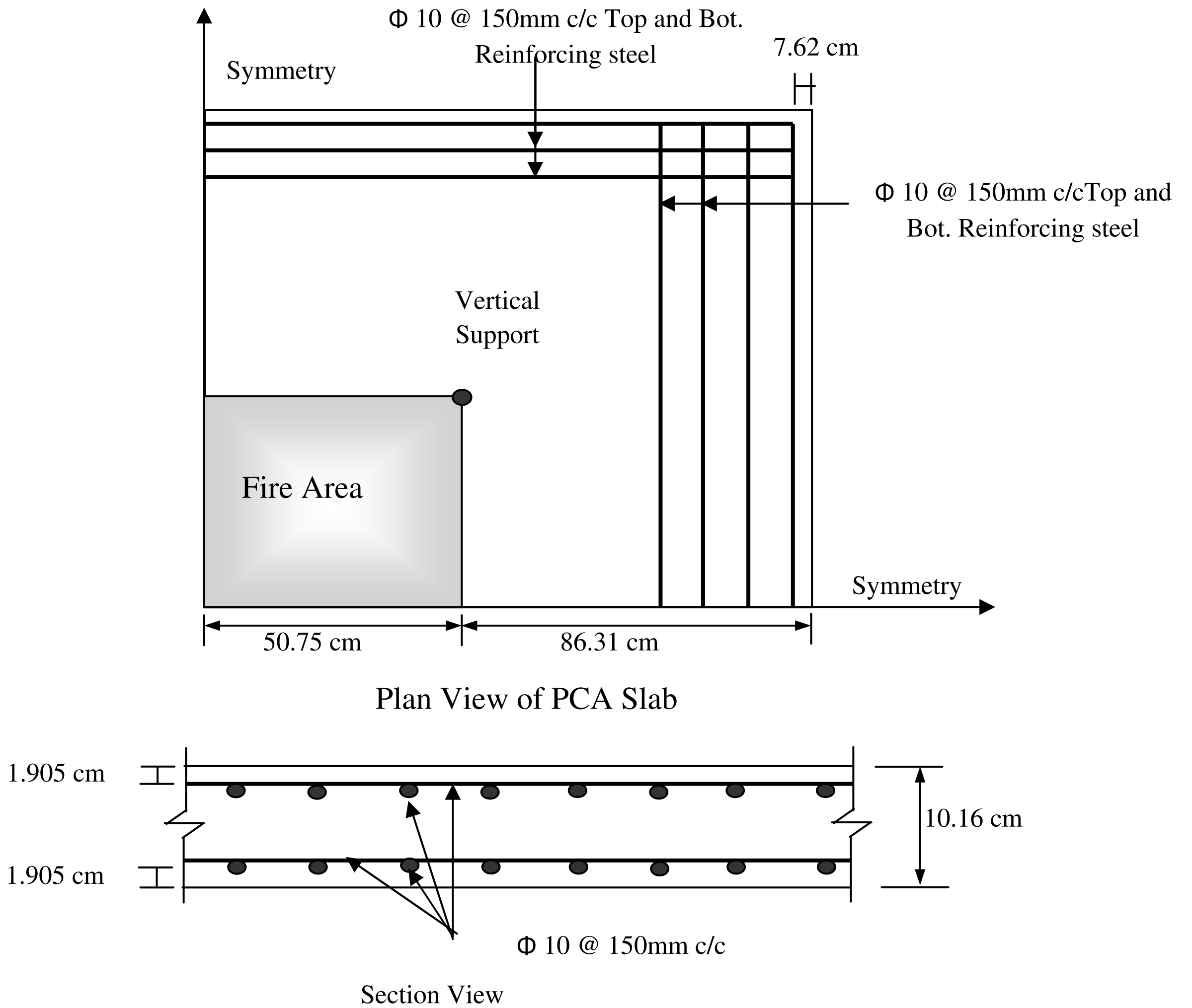

Figure 10. The slab analysed by Brost and Peeters.

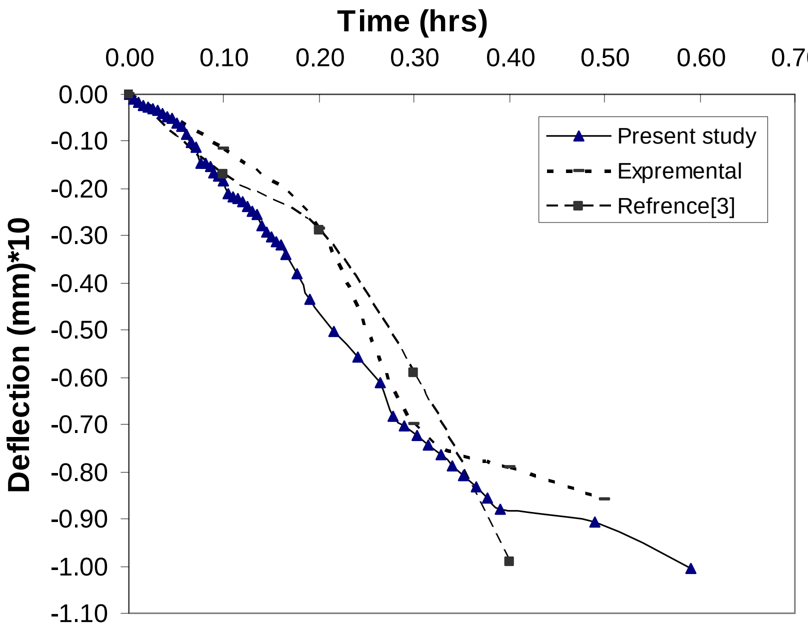

Figure 11. Experimental and calculated time-central deflection of reinforced concrete slab.

5. *Conclusion*

In this paper models and algorithms have been presented for analysis of reinforced concrete slabs at elevated temperatures. Models of concrete and steel behavior at elevated temperature are adopted from experimental studies of previous researches. These models yet need to be further studied and developed especially concrete biaxial failure envelope, concrete tensile strength, and Poisson ratio. The present model is verified with the available experimental tests, and the differences between the results could be attributed to the fact that structural fire testing even under laboratory conditions, is subjected to considerable uncertainty, it is difficult to determine the growth of fire and its temperature distribution at any time within the structure elements, control of the applied load, the degree of restraint at supports, concrete and steel degradation with temperature rise, however, even with the above mentioned factors, the numerical models give reasonable prediction of reinforced concrete slabs behaviour at elevated temperature.

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