Hydrodynamic Modeling of Porous Media; Analytical model of Water Filtration

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الخلاصة

لغرض نمذجة الجريان اللزج في الأوساط المسامية تم اختبار أربع نماذج من حشوات المرشحات المنتقاة و الشائعة الاستخدام صناعيا" والمتمثلة بالحبيبات المتكسرة من (سيلكا الرمل،الفحم،خبث الزجاج و الغرانيت)، حيث تم إيجاد العلاقات التجريبية لأنماط مختلفة من الجريان لتخمين الضائعات في المرشحات المستخدمة.

تم دراسة المتغيرات المؤثرة على قيم الخسائر في الأوساط المسامية (درجة الحرارة (20-80 مْ)، المسامية (35%-60%) وحجم الحبيبات (0.5-2 ملم). شملت الدراسة التحليلية استخدام التشابه البعدي واشتقاق معادلات الجريان المضطرب في الأوساط المسامية من الأسس الهايدروليكية.

تم استنتاج معادلة تجريبية لخسائر الطاقة في الاوساط المسامية وتقدير معامل الاحتكاك كدالة لرقم رينولد بمحاكاة الجريان في الانابيب.

Abstract

Hydrodynamic modeling of viscous flow in porous media was investigated for four selected filter media crushed silica, crushed anthracite coal, glass beads and crushed garnet. Typical constants that can be used to estimate head loss for some of the most common design of granular media filters were correlated. The effect of several parameters such as porosity (35%-60%) , temperature(20^{o}C-80^{o}C) and media grain size (0.5-2mm) was studied. Empirical relationships were developed using a plot of friction vs. Reynolds number similar to those that had been successfully used for the flow of fluids in pipes. Analytical models were made to develop an equation for viscous turbulent flow in porous media from first hydraulic principles. Empirical equation was developed to predict pressure drop in porous media as a function of bed porosity and evaluated the friction factor as a function to flow type.

1- Introduction

The steady flow through a bed of filter sand of various thickness and under various pressure is directly proportional to the hydraulic gradient. The Darcy's equation in the following from is commonly used (except at high velocities when turbulence occurs):

$$\Delta H / \Delta L = \frac{1}{K}\frac{Q}{A} \quad \text{--} \quad (1)$$

Where ΔH : head loss through medium (L), ΔL : depth of filter bed (L),
Q : flow rate through medium (L^3/T), A : area of the filter bed in plan (L^2) and K: Darcy's coefficient(L/T).

The hydrodynamics studies were led firstly by hydro-geologists, petroleum engineers and civil engineers interested in characterizing the flow of various fluids (water, oil and gas) in the underground environment. Mechanical and Chemical engineers interested in predicting the head loss in engineered media beds, specify the characteristics of the materials that make up the porous bed such as (water and air filter, cooling tower, scrubbers, absorber columns, ion exchange columns and evaporative beds).

For laminar flow (low flow velocity in porous media), Poiseuilles equation[1] can be applied to estimate the head loss from the velocity of flow in each individual capillary as follows:

$$\Delta H / L = 32\left(\frac{m}{rg}\right)\frac{V_c}{d_c^{\,2}} \quad \text{--} \ (2)$$

$$V_c = \frac{Q}{nA_b} = \frac{V.z}{n} \quad \text{--} \ (3)$$

and $$n = \frac{\forall_v}{\forall} \quad \text{---} \ (4)$$

Where L: length of capillary (L), m : dynamic viscosity (M/L.T), r : density (M/L^3), g: standard acceleration of gravity,(9.81m/**s²**)**,**d_c: inside diameter of capillary(L) and Vc: mean velocity in pores (L/T)**.**

n : bulk porosity of bed of porous media (L^3/L^3), $\forall_v$: volume of the voids (L^3), $\forall$: total volume of the bed (L^3), z : tortuosity (L/L).

The simple linear relationship between flow and head loss by Darcy has been demonstrated by Forchheimer[2] for condition under which the relationship between flow and head loss in porous media doesn't follow. Also notable were Kozeny[3] and White[4] applied similar principles to characterize the nonlinear resistance of porous media to the flow of gases at high Reynolds number. Nutting[5] defined the specific resistance and Wyckoff[6] popularized Nuttings specific resistance parameter, resulting in its wide use whenever the modeling of the flow of fluids underground was undertaken. Fancher[7], Fair and Hatch[8], Wallis[9] developed a powerful model for predicting Darcy resistance from the characteristics of the porous media. Many experimenters have attempted to use Reynolds concept to determine the upper limit of the validity of Darcy's law[10,11,12,13,14].

Various researchers have worked to show that forchheimers nonlinear equation could derived from the first principles beginning with the Navier-Stoikes equation[15,16,17,18,19,20,21,22,23]. Recently, modeling progresses of flows within unconsolidated, granular media rely mostly on experimental works using homogeneous, spherical, artificial media[24].In civil engineering, these researches are applied to the study of internal flows within earth and rock structures[25,26,27,28,29,30,31] and to the problems of similarities of flow parameters in centrifuged geotechnical small-scale models, within which very large hydraulic gradients are often found[32,33]. The extrapolation of these models to wide particle-size distributed natural media requires, there fore, specific studies.

Few studies have been conducted to examine post-Darcy flows within sands [32,34]. While most are restricted to macroscopic characteristics (permeability) and can be applied, consequently, only to the sands studied by the authors.

2- ***Experimental set-up and method***

The schematic diagram and the photograph of the experimental set-up model are presented in Figs. (1) and (2), respectively, consists of both a hydraulic device and a measurement device .The hydraulic device consists of a pump inverting the flow of water from a 400-liter tank through the material placed in the testing cell. When flowing out of the cell, water is forced back into the tank. The temperature is monitored at a quasi-constant value: 24-26 °C. Temperature sensors are placed in the tank, at the entrance and at the exit of the test cell .Then the values of the water density, r ,and of the water viscosity ,µ,are quasi constant during the tests(r =998.2kg/m^{3}and µ =1.008 Pa s).

Some valves are used to select the pressure tapings and the differential pressure gauge suiting the best measurement range. The measurement bench consists of three flow meters and four differential pressure gauges regulated by a valve set. The different measurement ranges of these devices are complementary and can be used to cover all the measurements accurately. The sand was packed as homogeneously as possible using the eluviations method with a constant 1.4 m height of fall. Satisfactory repeatability of the bulk densities was achieved throughout the experiments .The bed porosity is calculated knowing the cell volume and the mass of sand .

3- ***Analytical model in porous media at high flow velocity***

Solving practical filter design problems in hydrodynamic phenomena usually requires both theoretical and experimental results. By grouping significant quantities into dimensional parameters it is possible to reduce the number of variables appearing and to make this compact results (equation or chart) applicable to all similar situation.

Investigations consider the bed of porous media as composed of a number of capillary pores, passing in parallel, straight through the depth of the bed .In this case the head loss in high flow velocity (turbulent flow) depend upon capillary water velocity (V_c), dynamic viscosity (m), density (r). Using dimensional analysis (Buckingham Π theorem), general form of the empirical relationship can be construed as follow:

$$\frac{\Delta H}{L} = \Psi(\text{Re})\frac{1}{d_c}\frac{V_c^{\,2}}{2g} \text{ -- (5)}$$

$$Re = \frac{rVd}{m} \text{ -- (6-a)}$$

and $$Re_{pore} = \frac{4z \quad r \quad V}{m \quad (1-n) \quad S} \text{ -- (6-b)}$$

Where $\Psi(\text{Re})$ is a function to Reynolds number (Re) which is one of the most important dimensionless parameters in high flow velocity and its size determines the nature of flow. Turbulent flow in pipe, friction factor can be evaluated from (Blasius formula):

for $3x10^3 < Re<2x10^5$

$$f=\frac{0.316}{Re^{0.25}} \text{ -- } (7)$$

Similar to (Blasius formula) the hydraulic resistance factor (C_f) in porous media can be simulated as function to friction factor and as the form:

$$C_f = \frac{C'_f}{Re^{.25}} \text{ -- } \mathbf{(8)}$$

Where C'_f : constant depends upon porosity(n),uniform factor (Φ),and grain shape of media. Fig.(3)

The modeling approach presented here is centered on the characterization of macroscopic parameters (pore diameter , tortuosity and porosity of the medium, etc.),on the one hand, and on the determination of the range of validity of Darcy linear law based on a unique criterion common to all sands, on the other hand. The capillary model applied here has been developed by Comiti(35) and was first used for artificial, porous media (packing of spheres and plates).For laminar flow, friction factor can be evaluated as :

$$f_{pore}=\frac{64}{Re}+0.194 \quad , (Re \leq 4.3) \text{ -- } \mathbf{(9)}$$

Because of extreme complexity of grains shape of the media and its arrangement (spacing between the grains) much of the advance in understanding the basic relation has been developed around experiments on grains shape or specific surface (S_v) which is defined as:

$$S_v=\frac{S_m}{L_b A_b (1-n)} \text{ -- } (10)$$

For circular capillary tube

$$S_v=\frac{4n}{d_c(1-n)} \text{ -- } (11)$$

$$\forall_b = L_b A_b (1-n) \text{ -- } (12)$$

Where S_m : surface are of the media (L^2) and $\forall_b$: volume of the bed (L^3).

Equation (13) can be written after substituting (6,7,8,11) into Eq.(5).

$$\frac{\Delta H}{L} = C'_f \left(\frac{m}{r}\right)^{0.25} \left\{\frac{(1-n)^{1.25}}{n^3}\right\} S_v^{\ 1.25} z^{\ 1.75} V^{1.75} \text{ ---------------------------- } (13)$$

From the experimental data C'_f is (0.562 ∓ 2.4%) for ($1 \leq Re \leq 5$),(Laminar flow with increasing inertial influence), and C'_f is 0.349 ± 4.16% for ($5 \leq Re \leq 10$), (Inertial flow with increase ng random irregular flow), and C'_f for Turbulent flow ($10 \leq Re \leq 100$) is evaluated from Fig.(3).

$$C_f = \frac{0.7349}{Re^{0.25}} \text{ -- } \mathbf{(14)}$$

Fig.(1):The column test.

Fig.(2): The experimental filter model.

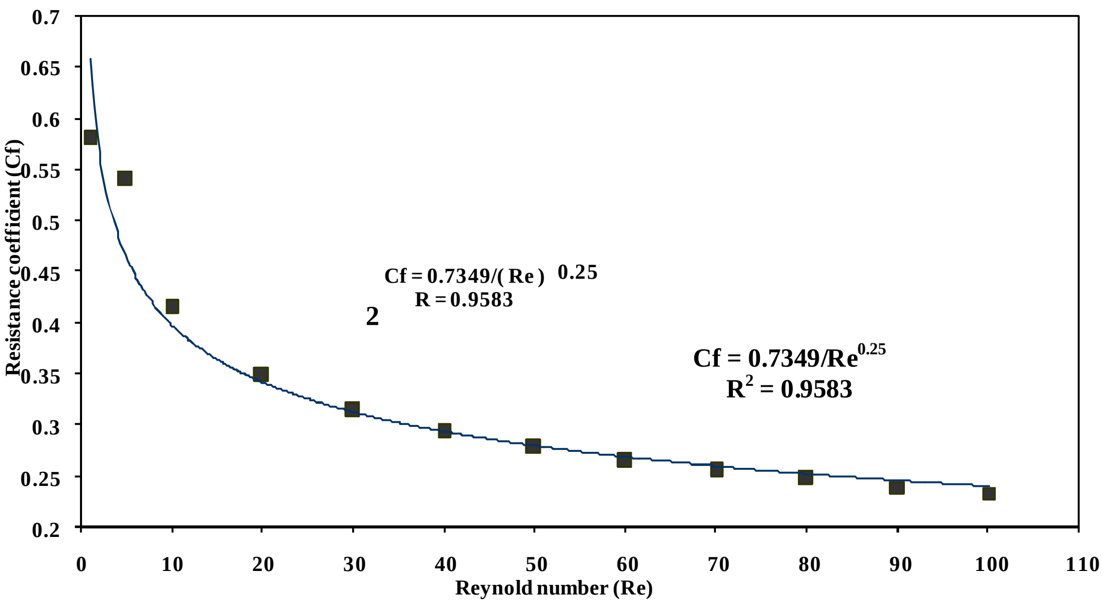

Fig.(3):- Resistance coefficient as a function to Reynold number in porous media.

4- Conclusion

1- Hydrodynamic modeling for viscous flow in porous media was investigated to estimate head loss in water filter.

2- Using dimensional analysis (Buckingham Π theorem) and the basic relation around experiments on grains shape or specific surface (Sv),porosity(n), tortuosity (z) were developed and empirical equation of head loss in porous media was evaluated as follow:

$$\frac{\Delta H}{L} = C'_f \left(\frac{m}{r}\right)^{0.25} \left\{\frac{(1-n)^{1.25}}{n^3}\right\} S_v^{\ 1.25} z^{\ 1.75} V^{1.75}$$

3- Empirical equation of friction factor in porous media as function to flow type can be evaluated as:

$$C_f = \frac{0.7349}{\mathrm{Re}^{0.25}}$$

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